$$\therefore dy = \frac{k'^2 z dz}{(1-k'^2 z^2)^{\frac{3}{2}}},\quad \frac{1}{\sqrt{[(1-y^2)(1-k^2y^2)]}} = \frac{\sqrt{(-1)}(1-k'^2z^2)}{k'^2 z\sqrt{(1-z^2)}}.$$

$$\therefore \int_1^{1/k} \frac{dy}{\sqrt{[(1-y^2)(1-k^2y^2)]}} = \sqrt{(-1)} \int_0^1 \frac{dz}{\sqrt{[(1-z^2)(1-k'^2z^2)]}}$$

$$= \sqrt{(-1)} F^I(k').$$

$$\therefore L = 2F^I(k) + \sqrt{(-1)} F^I[\sqrt{(1-k^2)}].$$

Also solved by the Proposer.

246. Proposed by C. N. SCHMALL, 89 Columbia Street, New York City.

Derive Taylor's Series by the use of the formula for successive integration by parts, and nothing else.

Solution by the PROPOSER.

Assume the identity, $F(a+h)-F(a)=\int_a^{a+h} F'(x)\,dx$, which on integrating the right hand member by parts,

$$-\Big[(a+h-x)F'(x)\Big]_a^{a+h} + \int_a^{a+h} (a+h-x)F''(x)\,dx$$

$$= hF'(a) - \Big[\frac{1}{2\,!}(a+h-x)^2 F''(x)\Big]_a^{a+h} + \frac{1}{2!}\int_a^{a+h}(a+h-x)^2 F'''(x)\,dx$$

$$= hF'(a) + \frac{h^2}{2\,!}F''(a) + \ldots + \frac{h^n}{n\,!}F^n(a) + \frac{1}{n!}\int_a^{a+h}(a+h-x)^n F^{n+1}(x)\,dx.$$

Hence, $F(a+h) - \Big[F(a) + h\,F'(a) + \frac{h^2}{2\,!}F''(a) + \ldots + \frac{h^n}{n\,!}F^n(a)\Big]$

$$(=R) = \frac{1}{n\,!}\int_a^{a+h}(a+h-x)^n F^{n+1}(x)\,dx.$$

We have assumed here that $F(x)$, $F'(x)$, $F''(x)$, ..., $F^n(x)$, $F^{n+1}(x)$, are all finite and continuous between the limits a and $(a+h)$ of x; now, if we put $A=F^{n+1}(a+\theta h)$, a mean value of $F^{n+1}(x)$ between the limits a and $(a+h)$, we have

$$R = \frac{1}{n!}\int_a^{a+h}(a+h-x)^n A\,dx = \frac{h^{n+1}}{(n+1)\,!}F^{n+1}(a+\theta h).$$

Also solved by Francis Rust.